Lesson 6 - Solving Quadratic Equations by Factoring. Discriminant. Quadratic Formula

Grade 10

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Quadratic Equations

A quadratic equation is an equation of the form $ax^2 + bx + c = 0$
where a, b, c are some numbers, $a \neq 0$ and x is the unknown.
a is called a leading coefficient and c is called a constant.

For example, in the equation $-3x^2 + 2x - 7 = 0$

$$a = -3 \quad b = 2 \quad c = -7$$

A quadratic equation, whose left side is a quadratic expression with a leading coefficient that's = 1 is called a simple trinomial. If the leading coefficient is not 1, we can always divide every part of the equation by the existing leading coefficient and make it equal to 1.

Sometimes terms in the equation might be "missing" (when *b* or *c* are = 0)

$$-5x^2 = 0; \quad 5x^2 - 45 = 0; \quad 3x^2 + 2x = 0$$

Quadratic Equations

The number of solutions when solving an equation where *b* or *c* = 0 can be determined as follows:

The equation of the type $ax^2 = 0$ has one solution $x = 0$

The equation of the type $ax^2 + c = 0$ can be rearranged into $x^2 = -\frac{c}{a}$ where if $-\frac{c}{a} < 0$ there are no solutions and

if $-\frac{c}{a} > 0$ there are two possible solutions: $x_1 = -\sqrt{-\frac{c}{a}}$ $x_2 = \sqrt{-\frac{c}{a}}$

The equation of the type $ax^2 + bx = 0$ can be common factored into $x(ax + b) = 0$; it has two possible roots: $x_1 = 0$ $x_2 = -\frac{b}{a}$;

x_2 results from a linear equation $ax + b = 0$

Solving Quadratic Equations by Factoring

When it is a simple trinomial equation $x^2 + px + q = 0$

then $x_1 + x_2 = -p, \quad x_1 \cdot x_2 = q$

When it is a complex trinomial equation $ax^2 + bx + c = 0$

then use the decomposition method to factor it

Quadratic Formula

When you are not able to factor, use the Quadratic Formula

Here is where the formula comes from (the proof):

$$
\begin{aligned}
ax^2 + bx + c &= 0 \\
4a^2x^2 + 4abx + 4ac &= 0 \\
4a^2x^2 + 4abx &= -4ac \\
4a^2x^2 + 4abx + b^2 &= b^2 - 4ac \\
(2ax + b)^2 &= b^2 - 4ac \\
2ax + b &= \pm\sqrt{b^2 - 4ac} \\
2ax &= -b \pm \sqrt{b^2 - 4ac} \\
\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}&.
\end{aligned}
$$

Discriminant

The Discriminant is derived from a Quadratic Formula and helps determine how many solutions the equation has.

$$D = b^2 - 4ac$$

If $D < 0$ then the equation has no real solutions

If $D = 0$ then the equation has one solution $x = -\frac{b}{2a}$ and the equation contains a perfect trinomial

If $D > 0$ then the equation has two possible solutions $x_1 = \frac{-b - \sqrt{D}}{2a}, \quad x_2 = \frac{-b + \sqrt{D}}{2a}$

Example: How many solutions does $x^2 - 6x - 13 = 0$ have?

$$D = 6^2 - 4 \cdot 1 \cdot 13 = 36 - 52 = -16$$

The equation has no real solutions, since $D < 0$

Solve a Quadratic Equation

$x^2 + 2x - 35 = 0$

$3x^2 + 5x + 2 = 0$

MOVE ON TO GRADE 10 LESSON 7