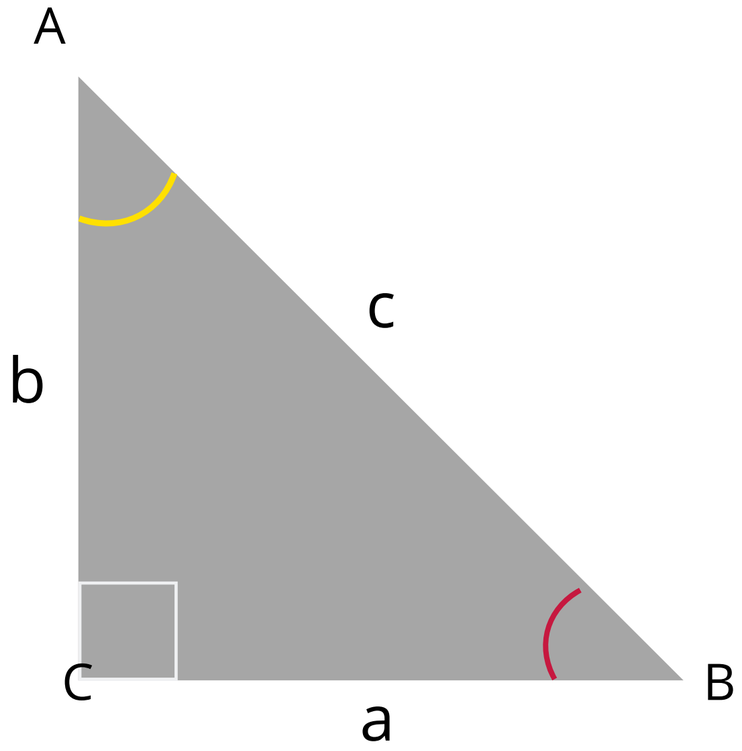
Lesson 5 - Trigonometry. Pythagorean Theorem

Grade 10

WWW.INTOMATH.ORG

RIGHT TRIANGLES



Let ABC be the right triangle, with C being the right angle.

CA and CB are LEGS
AB is a HYPOTENUSE (the longest, slant side)

Angles A and B are acute angles.
we can determine their measures by setting up
TRIGONOMETRIC RATIOS between sides and angles:
sine, cosine and tangent

$$\sin B = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{AC}{AB}$$

$$\cos B = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{CB}{AB}$$

$$\tan B = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{AC}{CB}$$

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

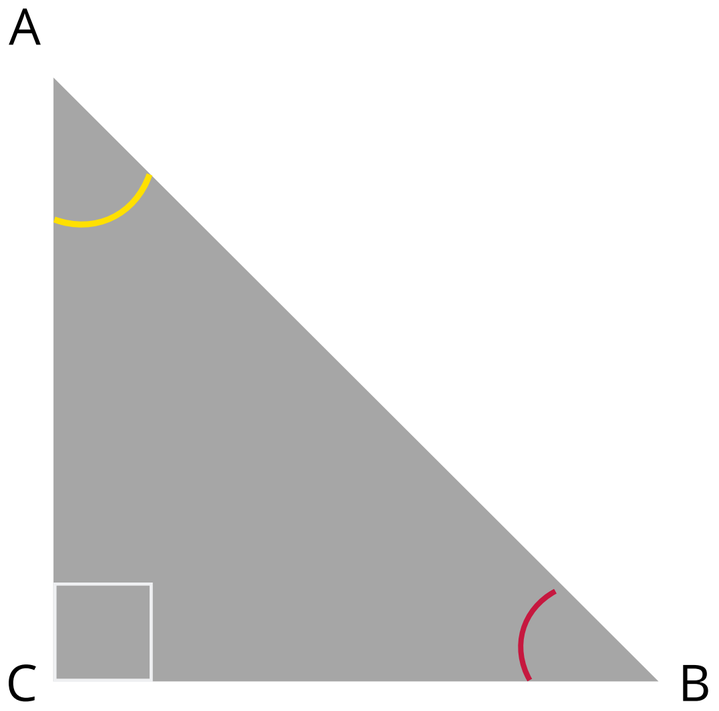
$$c^2 - b^2 = a^2$$

$$c^2 - a^2 = b^2$$

TRIGONOMETRIC RATIOS

SIN, COS and TAN depend only on the measure of the angle. Every acute angle has one possible value that represents the ratio of sin, cos or tan based on the given angle.

This is why the relationship between the measure of the angle and the value of the corresponding trigonometric ratio is a function (for every input there is only one possible output).



$$\sin B = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{AC}{AB}$$

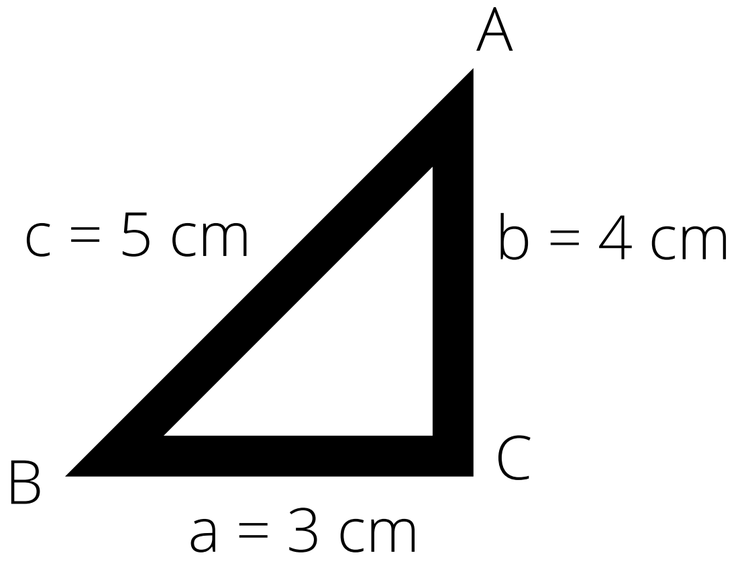
$$\cos B = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{CB}{AB}$$

$$\tan B = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{AC}{CB}$$

Example

The side lengths of the triangle are: 3 cm, 4 cm and 5 cm.
These side lengths form an Egyptian triangle, which is a right triangle, since $3^2 + 4^2 = 5^2$

If $\angle B = \beta$ determine $\sin\beta$ $\cos\beta$ $\tan\beta$

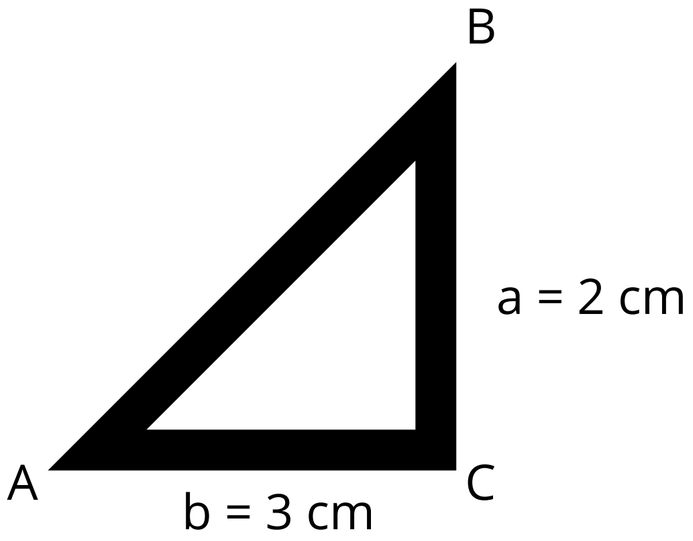


Solution:

$$\sin\beta = \frac{b}{c} = \frac{4}{5}, \qquad \cos\beta = \frac{a}{c} = \frac{3}{5}, \qquad \tan\beta = \frac{b}{a} = \frac{4}{3}$$

Solving for an Angle

In the follwoing right triangle determine the measure of angle B



Basic Trigonometric Identities

In mathematics, an **identity** is an equality relating one mathematical expression A to another mathematical expression B, such that A and B (which might contain some variables) produce the same value for all values of the variables within a certain range of validity.

The following identities are true for any angle:

$$\boxed{\sin^2\alpha + \cos^2\alpha = 1}$$

$$\boxed{\tan\alpha = \frac{\sin\alpha}{\cos\alpha}}$$

$$\boxed{1 + \tan^2\alpha = \frac{1}{\cos^2\alpha}}$$

$$\boxed{1 + \frac{1}{\tan^2\alpha} = \frac{1}{\sin^2\alpha}}$$

EXAMPLE

In a right triangle, determine the value of $\cos\alpha$ and $\tan\alpha$ if $\sin\alpha = \frac{12}{13}$

Based on the Pythagorean identity $\sin^2\alpha + \cos^2\alpha = 1$

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\left(\frac{12}{13}\right)^2} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

Based on the tangent identity $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{12}{13} \div \frac{5}{13} = \frac{12}{5}$$

Special Angles

α	30°	45°	60°
$\sin\alpha$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\alpha$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan\alpha$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

watch: **https://www.youtube.com/watch?v=LQ9z3iZiglg**

MOVE ON TO GRADE 10 LESSON 6